STRUCTURE OF WEAK SHOCK WAVES IN RELAXING MEDIA

G. M. Arutyunyan

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We examine the structure of a weak shock wave in media with temperature relaxation. It is assumed that the state of the matter in the transition layer changes slowly in relation to the establishment of equilibrium.

It is known that if a shock wave propagates through a relaxing medium and the relaxation time is significantly longer than the molecular collision time we observe a significant change of the shock wave structure and, in particular, a considerable increase of the width of its transitional layer. This question was first examined by Zel'dovich in [1] and later in other studies (see, for example [2]).

For strong and moderate shock waves the behavior of the parameters in the relaxation zone can be studied by numerical integration of the gasdynamic equations together with the kinetic equation. An example of such a numerical calculation in application to excitation of the vibrational degrees of freedom in molecular hydrogen is given in [3]. As for shock waves of low intensity, in this case an analytic solution of the problem is possible. Such a solution was given by D'yakov in [4] for the case in which the limiting values of the speed of sound differ very little from one another.

1. Assume the state of the matter in the transition layer of a shock wave changes slowly in relation to the establishment of equilibrium (for this the amplitude of the shock wave must be sufficiently small). Then, as shown in [4], the treatment of the relaxation process in the spirit of the Mandel'shtam-Leontovich method [5] can be used to study the shock wave structure. In accordance with this method, the presence of the relaxation processes for sufficiently slow changes of state is equivalent to an anomalously large second viscosity of the medium

$$\zeta = \tau\rho\,(u_\infty^2 - u_0^2) \tag{1.1}$$

Here τ is relaxation time, ρ is density of the medium, u_∞ is the speed of sound for frequencies which are so high that the relaxational processes in the sound wave are completely "frozen," u_0 is the speed of sound for frequencies which are so low that the medium in the sound wave can reach complete thermodynamic equilibrium. Consequently, in this case we can use the conventional formulas of hydrodynamic theory of the shock wave transition layer [6], in which heat conduction and the first viscosity are neglected.

Then we have the known formula for the behavior of the pressure in the transition layer

$$p = \frac{p_1 + p_0}{2} + \frac{p_1 - p_0}{2}\,\mathrm{th}\,\frac{x}{\delta} \tag{1.2}$$

Here $p_1 - p_0 = \Delta p$ is the pressure differential in the shock wave, and δ is the width of the transition zone, equal to

$$\delta = \frac{2V^3\zeta}{u_0^3(\partial^2 V/\partial p^2)_s\,\Delta p}\,. \tag{1.3}$$

In (1.3) V is specific volume, ζ is defined by (1.1), and s is entropy.

Since these formulas are applicable when the state of the medium in the transition layer varies slowly in relation to the establishment of equilibrium, this means that

$$\delta \gg u_0\tau\,. \tag{1.4}$$

Using (1.1) and (1.3), we write (1.4) as

$$\Delta p \ll \frac{2V^2(u_\infty^2 - u_0^2)}{u_0^4 (\partial^2 V / \partial p^2)_s} \quad . \tag{1.5}$$

This is the formulation in general form for applicability of the second viscosity concept.

In the present paper we examine the structure of a weak shock wave in media in which there is temperature relaxation. It is assumed that the state of the matter in the transition layer changes slowly in relation to the establishment of equilibrium, i.e., condition (1.5) is satisfied and therefore (1.2) and (1.3) are applicable. As the relaxing media we examine two-component mixtures, such as emulsions of one liquid in another or a gas and suspension of solid particles in it. During compression of the matter in the shock wave, a temperature difference develops between the components of such a medium and this difference leads to the heat transfer relaxation process. (We note that when (1.3) and (1.1) are substituted into (1.4), τ is excluded from (1.5). Since of all the quantities appearing in these relations only τ depends on the thermal conductivity of the suspension particles, the condition of applicability of the second viscosity concept (1.5) is independent of the thermal conductivity of these particles.)

We shall examine this question in more detail, assuming hereafter that the nonhomogeneity scale of the medium is small in comparison with the width of the shock wave transition layer.

2. We represent the relaxing medium as a system consisting of a filter (first component) with particles (second component) distributed in it. To determine the relaxation time τ we denote by r the radius of the particles suspended in the filler and examine, for example, the problem of propagation of periodic temperature oscillations in a homogeneous half-space. We examine this problem without initial conditions, since in the case of multiple repetition of the temperature cycle on the surface we can neglect the influence of the initial temperature of the medium. Thus we arrive at the problem [7]

$$\frac{\partial \Delta T}{\partial t} = a^2 \frac{\partial^2 \Delta T}{\partial x^2} \quad (0 \leqslant x < \infty, \ -\infty < t) \tag{2.1}$$

where ΔT satisfies the condition

$$\Delta T\ (0,\ t) = A\ \cos \omega t \tag{2.2}$$

Here ΔT is the difference between the absolute temperature of the point x at time t and its initial temperature, a^2 is the thermal diffusivity of the medium, and ω is the frequency of the temperature oscillation at its surface.

The solution of this problem has the form [8]

$$\Delta T(x,\ t) = A \exp\left(-\sqrt{\frac{\omega}{2a^2}}\, x\right) \cos\left(\sqrt{\frac{\omega}{2a^2}}\, x - \omega t\right). \tag{2.3}$$

It follows from (2.3) that if the surface temperature changes periodically, temperature oscillations with the same period are also established in the medium, and the amplitude of the oscillations decreases exponentially with depth:

$$A(x) = A\ \exp\left(-\sqrt{\frac{\omega}{2a^2}}\, x\right). \tag{2.4}$$

Let us determine the period of the temperature oscillations at the surface for which the amplitude of the oscillation diminishes by a factor of e at the distance r. The magnitude of this period can obviously be taken equal to the unknown relaxation time. Assuming, in addition, that $a^2 = \varkappa/\rho c$ (where $\varkappa$ is the thermal conductivity of the medium, c is its specific heat, and ρ is density), we obtain

$$\tau = \pi \rho c r^2 / \varkappa . \tag{2.5}$$

3. Knowing the mechanical and thermodynamic constants of the components of the medium, we can determine the limiting values of the speed of sound (we note that there is an obvious misprint in the formula for u_∞ presented in [9], which can be established easily on the basis of dimensional analysis). We use subscripts to denote quantities relating

to the first and second components, respectively; they are omitted on quantities relating to both the first and second components and to the mixture as a whole.

We have for the adiabatic compressibility of an individual component

$$-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = -\frac{1}{\gamma V}\left(\frac{\partial V}{\partial p}\right)_T \tag{3.1}$$

Using known thermodynamic relations [10], it is easy to show with the aid of (3.1) that

$$-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_s = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T - \frac{T}{\rho c_p V^2}\left(\frac{\partial V}{\partial T}\right)_p^2 \tag{3.2}$$

where c_p is the constant-pressure specific heat.

Now let us examine an arbitrary mass of the mixture of volume V. In determining u_∞ we must keep in mind that this assumes such rapid change of the state of the medium that there is no heat transfer between its components. In this connection it is well to introduce the concept of the "adiabatic-adiabatic" compressibility of the medium, which obviously will be

$$\sigma_{ss} = -\left(\frac{\partial V_1}{\partial p}\right)_s - \left(\frac{\partial V_2}{\partial p}\right)_s \tag{3.3}$$

Then we have for u_∞

$$u_\infty = \sigma_{ss}^{\,1/2} / V \tag{3.4}$$

where V now means the specific volume of the mixture. We substitute into (3.4) the value of σ_{ss} referred to unit mass of the medium and we make use of (3.1) and (3.2). Then, assuming that for the mixture

$$\rho = \varepsilon\rho_1 + (1-\varepsilon)\rho_2 \tag{3.5}$$

(ε is the volume concentration of the first component), we finally obtain

$$u_\infty^{-2} = -[\varepsilon\rho_1 + (1-\varepsilon)\rho_2]\left[\frac{\varepsilon\rho_1}{\gamma_1}\left(\frac{\partial V_1}{\partial p}\right)_T + \frac{(1-\varepsilon)\rho_2}{\gamma_2}\left(\frac{\partial V_2}{\partial p}\right)_T\right]. \tag{3.6}$$

In (3.6) V_1 and V_2 are the specific volumes of the components of the mixture.

In determining u_0 it is necessary to know the "adiabatic-isothermal" compressibility of the mixture. We denote it by σ_{TS}. This assumes such slow change of the state of the mixture that the "macroscopic" compression and expansion processes take place adiabatically, while the "microscopic" processes are isothermal, i.e., the temperature difference between the components can equalize. Consequently, in this case the components of the mixture are at all times at the same pressure and temperature. This means that in determining σ_{TS} we must take p and T as the independent thermodynamic variables. Using for this purpose relations (3.2) and taking account of the additivity of the quantities in this expression, we obtain

$$\sigma_{TS} = -\left(\frac{\partial V_1}{\partial p}\right)_T - \left(\frac{\partial V_2}{\partial p}\right)_T - \frac{T}{(V_1+V_2)[\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}]}\left[\left(\frac{\partial V_1}{\partial T}\right)_p + \left(\frac{\partial V_2}{\partial T}\right)_p\right]^2. \tag{3.7}$$

We have for u_0

$$u_0 = \sigma_{TS}^{1/2} / V \tag{3.8}$$

where, as in (3.4), V is the specific volume. Substituting the value of σ_{TS}, per unit mass into (3.8), we finally obtain

$$u_0^{-2} = -[\varepsilon\rho_1 + (1-\varepsilon)\rho_2]\left\{\varepsilon\rho_1\left(\frac{\partial V_1}{\partial p}\right)_T + (1-\varepsilon)\rho_2\left(\frac{\partial V_2}{\partial p}\right)_T + \right.$$
$$\left. + \frac{T}{\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}}\left[\varepsilon\rho_1\left(\frac{\partial V_1}{\partial T}\right)_p + (1-\varepsilon)\rho_2\left(\frac{\partial V_2}{\partial T}\right)_p\right]^2\right\}. \tag{3.9}$$

In (3.9) V_1 and V_2 are the specific volumes of the first and second components, and c_{p1} and c_{p2} are their specific heats.

4. The basis of this analysis is the assumption of slow change of the state of the medium in the transition layer in relation to the establishment of equilibrium. Consequently the quantity $(\partial^2V/\partial p^2)_s$, in (1.3) and (1.5) must be determined with account for the fact that the temperature difference between the components of the medium can equalize during compression in the shock wave, i.e., macroscopically the process takes place adiabatically, while microscopically it takes place isothermally. Thus, here again we must take the pressure and temperature as the independent variables.

To do this we represent the quantity $(\partial^2V/\partial p^2)_s$ in terms of the variables p and T for the individual component. It follows from (3.2) that

$$\left(\frac{\partial V}{\partial p}\right)_s = f(p, T) \tag{4.1}$$

where the function $f(p,T)$ denotes

$$f(p, T) = \left(\frac{\partial V}{\partial p}\right)_T + \frac{T}{c_p}\left(\frac{\partial V}{\partial T}\right)_p^2. \tag{4.2}$$

Considering T as a function of p and s, after differentiating (4.1) with respect to p we obtain

$$\left(\frac{\partial^2 V}{\partial p^2}\right)_s = \left(\frac{\partial f}{\partial p}\right)_T + \left(\frac{\partial f}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_s. \tag{4.3}$$

Setting $c_p = \text{const}$, it is easy to find on the basis of (4.2) the values of $(\partial f/\partial p)_T$ and $(\partial f/\partial T)_p$. Then, using the known thermodynamic relation [10]

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{T}{c_p}\left(\frac{\partial V}{\partial T}\right)_p \tag{4.4}$$

we finally obtain

$$\left(\frac{\partial^2 V}{\partial p^2}\right)_s = \left(\frac{\partial^2 V}{\partial p^2}\right)_T + \frac{3T}{c_p}\left(\frac{\partial V}{\partial T}\right)_p\frac{\partial^2 V}{\partial p\,\partial T} + \frac{T}{c_p^2}\left(\frac{\partial V}{\partial T}\right)_p^2\left[\left(\frac{\partial V}{\partial T}\right)_p + 2T\left(\frac{\partial^2 V}{\partial T^2}\right)_p\right]. \tag{4.5}$$

In (4.5) V and c_p are either the specific volume and the specific heat or the volume and heat capacity of an arbitrary mass.

Considering the additivity of the quantities appearing in (4.5), we can show that for unit mass of the mixture

$$\left(\frac{\partial^2 V}{\partial p^2}\right)_s = n\left(\frac{\partial^2 V_1}{\partial p^2}\right)_T + (1-n)\left(\frac{\partial^2 V_2}{\partial p^2}\right)_T + \frac{3T}{nc_{p1} + (1-n)c_{p2}}\left[n\left(\frac{\partial V_1}{\partial T}\right)_p + \right.$$
$$\left. + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p\right]\left[n\frac{\partial^2 V_1}{\partial p\,\partial T} + (1-n)\frac{\partial^2 V_2}{\partial p\partial T}\right] + \frac{T}{[nc_{p1} + (1-n)c_{p2}]^2}\left[n\left(\frac{\partial V_1}{\partial T}\right)_p + \right.$$
$$\left. + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p\right]^2\left\{n\left(\frac{\partial V_1}{\partial T}\right)_p + (1-n)\left(\frac{\partial V_2}{\partial T}\right)_p + 2T\left[n\left(\frac{\partial^2 V_1}{\partial T^2}\right)_p + (1-n)\left(\frac{\partial^2 V_2}{\partial T^2}\right)_p\right]\right\}. \tag{4.6}$$

In (4.6) V_1 and V_2 are the specific volumes of the components of the mixture, c_{p1} and c_{p2} are their specific heats, and n is the mass concentration of the first component, connected with ε by the relation

$$n = \frac{\varepsilon\rho_1}{\varepsilon\rho_1 + (1-\varepsilon)\rho_2} \tag{4.7}$$

5. Assume that the first component of the mixture is a gas and the second consists of solid or liquid particles suspended in the gas. For such systems we can neglect the compressibility of the second component in comparison with the first. This leads to considerable simplification of the relations obtained above. In fact, we set in (3.6), (3.9), and (4.6)

$$\left(\frac{\partial V_2}{\partial p}\right)_T = \left(\frac{\partial V_2}{\partial T}\right)_p = \left(\frac{\partial^2 V_2}{\partial p^2}\right)_T = \left(\frac{\partial^2 V_2}{\partial T^2}\right)_p = \frac{\partial^2 V_2}{\partial p\, \partial T} = 0 .$$

Then, using the equation of state of the ideal gas to calculate the derivatives, we obtain for u_∞

$$u_\infty = \left\{\frac{\rho_1}{\varepsilon[\varepsilon\rho_1 + (1-\varepsilon)\rho_2]}\right\}^{1/2} u_1 \tag{5.1}$$

and we obtain for u_0

$$u_0 = \left\{\frac{\rho_1[\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}]}{\varepsilon[\varepsilon\rho_1 + (1-\varepsilon)\rho_2][\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\gamma_1\rho_2 c_{p2}]}\right\}^{1/2} u_1 . \tag{5.2}$$

Here the quantity $(\partial^2 V/\partial p^2)_s$ will be defined by the formula

$$\left(\frac{\partial^2 V}{\partial p^2}\right)_s = \frac{\varepsilon[\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\gamma_1\rho_2 c_{p2}][\varepsilon(\gamma_1+1)\rho_1 c_{p1} + 2(1-\varepsilon)\gamma_1\rho_2 c_{p2}]}{\gamma_1^2 p^2[\varepsilon\rho_1 + (1-\varepsilon)\rho_2][\varepsilon\rho_1 c_{p1} + (1-\varepsilon)\rho_2 c_{p2}]^2} . \tag{5.3}$$

In (5.1)–(5.3) u_1 is the conventional speed of sound in the gas, corresponding to the first component of the medium, i.e.,

$$u_1 = \left(\frac{\gamma_1 p}{\rho_1}\right)^{1/2} . \tag{5.4}$$

Since $\gamma_1 > 1$, it follows immediately from (5.1) and (5.2) that in the case in question $u_\infty > u_0$ always.

The assumption made on incompressibility of the second component makes it possible to simplify considerably the form of condition (1.5) concerning applicability of the second viscosity concept. In fact, if in the right-hand side of (1.5) we substitute (3.5) and the formulas (5.1)–(5.3) obtained in this section, then after several transformations we can obtain the relation

$$\frac{\Delta p}{p} \ll \frac{2\gamma_1\rho_2 c_{p2}(\gamma_1 - 1)(1-\varepsilon)}{(\gamma_1+1)\rho_1 c_{p1}\varepsilon + 2\gamma_1\rho_2 c_{p2}(1-\varepsilon)} = \psi(\varepsilon) . \tag{5.5}$$

It follows from (5.5) that $\psi(0) = \gamma_1 - 1$ and $\psi(1) = 0$. In the interval $[0,1]\,\psi(\varepsilon)$ decreases monotonically, since

$$\frac{d\psi}{d\varepsilon} = -\frac{2\gamma_1\rho_1\rho_2 c_{p1} c_{p2}(\gamma_1^2 - 1)}{[(\gamma_1+1)\rho_1 c_{p1}\varepsilon + 2\gamma_1\rho_2 c_{p2}(1-\varepsilon)]^2} < 0 .$$

Consequently, the larger the gas content in the mixture the weaker the shock wave must be in order that the condition concerning applicability of the second viscosity be satisfied. However, we must note that on most of the interval $[0,1]$ the reduction of ψ is very slight and ψ is essentially equal to $\gamma_1 - 1$. Only for values of ε very close to unity does the change of ψ become significant.

In fact, it follows from (5.5) that

$$2\gamma_1\rho_2 c_{p2}\ (1-\varepsilon) \geqslant 10\,(\gamma_1+1)\,\rho_1 c_{p1}\,\varepsilon \gg (\gamma_1+1)\rho_1 c_{p1}\varepsilon \tag{5.6}$$

and therefore we can set $\psi(\varepsilon) = \gamma_1 - 1$, if

$$\varepsilon \ll \left(1 + \frac{5(\gamma_1 + 1)}{\gamma_1} \frac{\rho_1 c_{p1}}{\rho_2 c_{p2}}\right)^{-1}. \tag{5.7}$$

However, since on the other hand

$$\alpha = \frac{5(\gamma_1 + 1)}{\gamma_1} \frac{\rho_1 c_{p1}}{\rho_2 c_{p2}} \ll 1$$

by expanding the right-hand side of (5.7) into a series in α this condition can be written in the form

$$\varepsilon \ll 1 - \frac{5(\gamma_1 + 1)}{\gamma_1} \frac{\rho_1 c_{p1}}{\rho_2 c_{p2}} = \varepsilon_*. \tag{5.8}$$

Thus, if $\varepsilon \lesssim \varepsilon_*$, the condition for applicability of the second viscosity is written in the simple form

$$\Delta p / p \ll \gamma_1 - 1 . \tag{5.9}$$

Calculations showed that for systems such as helium-coal, air-coal, air-water, and air-quartz, the values of ε_* equal, respectively, 0.993, 0.990, 0.998, and 0.995.

$1-\varepsilon$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
			helium-coal		
u_∞	$3.78 \cdot 10^3$	$1.13 \cdot 10^4$	$3.37 \cdot 10^4$	$7.46 \cdot 10^4$	$9.55 \cdot 10^4$
u_0	$2.93 \cdot 10^3$	$8.89 \cdot 10^3$	$2.88 \cdot 10^4$	$7.2 \cdot 10^4$	$9.51 \cdot 10^4$
ψ	0.666	0.629	0.405	$8.88 \cdot 10^{-2}$	$1.01 \cdot 10^{-2}$
$\Delta p\delta$	$1.54 \cdot 10^3$	$4.42 \cdot 10^3$	$9.86 \cdot 10^3$	$6.73 \cdot 10^3$	$1.14 \cdot 10^3$
δ/δ_0	$1.68 \cdot 10^1$	$4.83 \cdot 10^1$	$1.08 \cdot 10^2$	$7.35 \cdot 10^1$	$1.24 \cdot 10^1$
			air-coal		
u_∞	$3.44 \cdot 10^3$	$9.98 \cdot 10^3$	$2.37 \cdot 10^4$	$3.24 \cdot 10^4$	$3.41 \cdot 10^4$
u_0	$2.92 \cdot 10^3$	$8.56 \cdot 10^3$	$2.17 \cdot 10^4$	$3.19 \cdot 10^4$	$3.39 \cdot 10^4$
ψ	0.396	0.366	0.205	$3.8 \cdot 10^{-2}$	$4.15 \cdot 10^{-2}$
$\Delta p\delta$	$1.04 \cdot 10^3$	$2.81 \cdot 10^3$	$4.02 \cdot 10^3$	$1.14 \cdot 10^3$	$1.35 \cdot 10^2$
δ/δ_0	$3.39 \cdot 10^1$	$9.17 \cdot 10^1$	$1.31 \cdot 10^2$	$3.72 \cdot 10^1$	4.4
			air-water		
u_∞	$3.92 \cdot 10^3$	$1.12 \cdot 10^4$	$2.52 \cdot 10^4$	$3.28 \cdot 10^4$	$3.4 \cdot 10^4$
u_0	$3.32 \cdot 10^3$	$9.54 \cdot 10^3$	$2.2 \cdot 10^4$	$3.12 \cdot 10^4$	$3.38 \cdot 10^4$
ψ	0.399	0.390	0.321	0.116	$1.56 \cdot 10^{-2}$
$\Delta p\delta$	$3.14 \cdot 10^4$	$8.84 \cdot 10^4$	$1.67 \cdot 10^5$	$8.74 \cdot 10^4$	$1.32 \cdot 10^4$
δ/δ_0	$1.02 \cdot 10^3$	$2.88 \cdot 10^3$	$5.45 \cdot 10^3$	$2.85 \cdot 10^3$	$4.31 \cdot 10^2$

The table presents the results of comparison of the width of the transition layer of a weak shock wave in pure helium and pure air, calculated on the basis of the usual formula of hydrodynamic theory of the transition layer [6] (in this formula both viscosities are assumed to have the same order of magnitude), with the width of the transition layer in the helium-coal, air-coal, and air-water systems, calculated on the basis of the relations obtained above. It was assumed that in all cases the dimension of the particle suspended in the gas was $r = 10^{-4}$ cm, $p = 10^6$ dynes/cm^2, $T = 288°$ K, and all the required data on the properties of the helium, air, coal, and water were taken from [11]. In this table δ is the width of the transition layer in the mixture, and δ_0 is the width of the transition layer in the corresponding gas for the same values of Δp. All the data are represented in the CGS system.

These data show that over a wide range of variation of $1 - \varepsilon$ the width of the transition layer in the mixture in the helium-coal and air-coal systems is on the average more than an order of magnitude greater than in pure helium and pure air. This difference is still greater in the air-water system, where the difference reaches three orders of magnitude in comparison with the pure air.

As the content of the solid and liquid particles in the mixture decreases the value of δ initially increases and after reaching a maximum then begins to decrease. These data also confirm the statements made above concerning the variation of the function $\psi(\varepsilon)$.

6. When the densities of the components of the mixture differ significantly from one another, i.e., when one of the components is a gas, the presence of the diffusion and barodiffusion phenomena leads to a change of the concentration of the components in the transition layer of the weak shock wave and results in a considerable increase of its width. As was shown in [12], in this case

$$\delta \sim \frac{\rho_2 c_{p1} l T (1-n)}{\Delta p}\left(1-\frac{1}{\gamma_1}\right)\left(\frac{r}{l}\right)^2, \tag{6.1}$$

where l is the mean free path of the gaseous molecule. Consequently, in such cases we must obviously consider in addition to the thermal relaxation mechanism this additional mechanism of increase of δ. However, it can be shown that the two processes are practically independent and do not affect one another.

In fact, it follows from the above table that a very large change of the concentration of the solid and liquid particles in the gas has very little effect on the magnitude of δ resulting from thermal relaxation. This effect will be even less important for weak shock waves, where the concentration changes are small. Since these changes are due to the diffusion and barodiffusion phenomena the latter will have practically no influence on the thermal relaxation process.

Conversely, it follows from (6.1) that small changes of the temperature in the transition layer of weak shock waves, which are connected with thermal relaxation, will not affect the value of δ due to the diffusion and barodiffusion phenomena.

Thus the processes in question actually are independent. This means that the width of the shock wave transition layer will be determined by the larger of the two values of δ owing to these processes separately.

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